



Universität zu Köln

Seminar für Allgemeine Betriebswirtschaftslehre, Supply Chain Management und Management Science

Primer on Inventory Management

Continuous Review Inventory Model

© Prof. Dr. Ulrich W. Thonemann

Continuous review inventory model

Assumptions

- At most one order is outstanding
- Excess demand is backordered
- Fixed order costs are relevant
- Demand is Normally distributed

Inventory policy

If the inventory level reaches the re-order point r , then order the quantity x

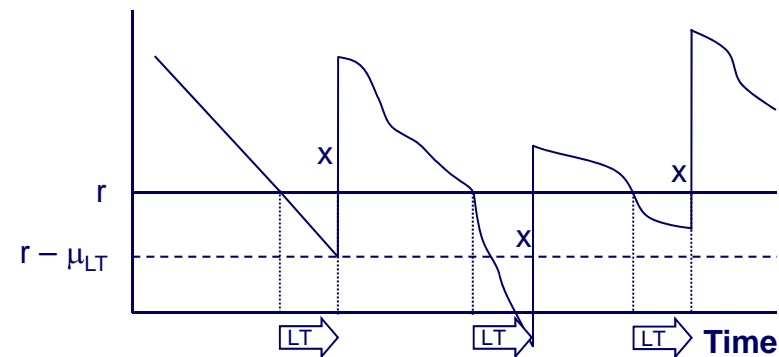
Parameters

- μ Mean demand (100 units/day)
- σ Standard deviation of demand (90 units/day)
- h Unit inventory holding cost (0.20 €/unit/year)
- p Unit backorder penalty cost (3.00 €/unit/period)
- K Fixed order cost (50 €/order)
- LT Lead time (9 days)

Decision variables

- r Re-order point
- x Order quantity

Inventory level



Expected inventory, backorders, and orders

$$E[\text{Inventory}] \approx r - \mu_{LT} + \frac{x}{2}$$

$$E[\text{Backorders per order cycle}] \approx \int_{y=r}^{\infty} (y^{LT} - r) f_{LT}(y^{LT}) dy^{LT}$$

$$= L \left(\frac{r - \mu_{LT}}{\sigma_{LT}} \right) \sigma_{LT}$$

$$E[\text{\#Orders}] = \frac{\mu}{x} \quad (\text{this is also the number of order cycles})$$

Optimal solution

Expected cost

$$Z(x,r) \approx h \left(r - \mu_{LT} + \frac{x}{2} \right) + p \frac{\mu}{x} L \left(\frac{r - \mu_{LT}}{\sigma_{LT}} \right) \sigma_{LT} + K \frac{\mu}{x}$$

First order condition

$$x^* = \sqrt{\frac{2\mu}{h} \left(K + pL \left(\frac{r^* - \mu_{LT}}{\sigma_{LT}} \right) \sigma_{LT} \right)} \quad r^* = F_{LT}^{-1} \left(1 - \frac{x^* h}{\mu p} \right)$$

Iterative solution approach

$$x_0 = \sqrt{\frac{2\mu K}{h}} = \sqrt{\frac{2 \cdot 36,500 \cdot 50}{0.20}} = 4,272 \text{ units}$$

$$r_0 = F_{LT}^{-1} \left(1 - \frac{x^* h}{\mu p} \right) = F_{LT}^{-1} \left(1 - \frac{4,272 \cdot 0.20}{36,500 \cdot 3} \right) \\ = F_{LT}^{-1}(0.9922) = 9 \cdot 100 + 2.42 \cdot \sqrt{9} \cdot 90 = 1,553 \text{ units}$$

$$x_1 = \sqrt{\frac{2 \cdot 36,500}{0.20} \left(50 + 3 \cdot L \left(\frac{1,553 - 900}{270} \right) \cdot 270 \right)} = 4,361 \text{ units}$$

$$r_1 = \dots = 1,551 \text{ units}$$

α -Service level constraint

The probability that all demand in an **order cycle** is met must be at least α (for example, $\alpha = 95\%$)

$$x^* = \sqrt{\frac{2\mu K}{h}} = \sqrt{\frac{2 \cdot 36,500 \cdot 50}{0.20}} = 4,272 \text{ units}$$

$$r^* = F_{LT}^{-1}(\alpha) = \mu_{LT} + z \sigma_{LT} = 100 \cdot 9 + 1.64 \cdot \sqrt{9} \cdot 90 = 1,343 \text{ units}$$

β -Service level constraint

The expected fraction of the demand that is filled in an order cycle must be at least β (for example, $\beta = 99\%$)

$$x^* \approx \sqrt{\frac{2\mu K}{h}} = \sqrt{\frac{2 \cdot 36,500 \cdot 50}{0.20}} = 4,272 \text{ units}$$

$$\beta \approx \frac{x^* - L \left(\frac{r^* - \mu_{LT}}{\sigma_{LT}} \right) \sigma_{LT}}{x^*} \Leftrightarrow r^* = \mu_{LT} + L^{-1} \left(\frac{(1-\beta)x^*}{\sigma_{LT}} \right) \sigma_{LT}$$

$$r^* = 9 \cdot 100 + L^{-1} \left(\frac{(1-0.99) \cdot 4,272}{\sqrt{9} \cdot 90} \right) \sqrt{9} \cdot 90 = 1,073 \text{ units}$$

$\underbrace{\hspace{10em}}_{0.64}$

Appendix

—

Table of the Standard Normal Distribution

| z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) |
|----------|-------------|-------------|-------------|----------|-------------|-------------|-------------|----------|-------------|-------------|-------------|----------|-------------|-------------|-------------|
| -3.00 | 0.0044 | 0.0013 | 3.0004 | -2.50 | 0.0175 | 0.0062 | 2.5020 | -2.00 | 0.0540 | 0.0228 | 2.0085 | -1.50 | 0.1295 | 0.0668 | 1.5293 |
| -2.99 | 0.0046 | 0.0014 | 2.9904 | -2.49 | 0.0180 | 0.0064 | 2.4921 | -1.99 | 0.0551 | 0.0233 | 1.9987 | -1.49 | 0.1315 | 0.0681 | 1.5200 |
| -2.98 | 0.0047 | 0.0014 | 2.9804 | -2.48 | 0.0184 | 0.0066 | 2.4821 | -1.98 | 0.0562 | 0.0239 | 1.9890 | -1.48 | 0.1334 | 0.0694 | 1.5107 |
| -2.97 | 0.0048 | 0.0015 | 2.9704 | -2.47 | 0.0189 | 0.0068 | 2.4722 | -1.97 | 0.0573 | 0.0244 | 1.9792 | -1.47 | 0.1354 | 0.0708 | 1.5014 |
| -2.96 | 0.0050 | 0.0015 | 2.9604 | -2.46 | 0.0194 | 0.0069 | 2.4623 | -1.96 | 0.0584 | 0.0250 | 1.9694 | -1.46 | 0.1374 | 0.0721 | 1.4921 |
| -2.95 | 0.0051 | 0.0016 | 2.9505 | -2.45 | 0.0198 | 0.0071 | 2.4523 | -1.95 | 0.0596 | 0.0256 | 1.9597 | -1.45 | 0.1394 | 0.0735 | 1.4828 |
| -2.94 | 0.0053 | 0.0016 | 2.9405 | -2.44 | 0.0203 | 0.0073 | 2.4424 | -1.94 | 0.0608 | 0.0262 | 1.9500 | -1.44 | 0.1415 | 0.0749 | 1.4736 |
| -2.93 | 0.0055 | 0.0017 | 2.9305 | -2.43 | 0.0208 | 0.0075 | 2.4325 | -1.93 | 0.0620 | 0.0268 | 1.9402 | -1.43 | 0.1435 | 0.0764 | 1.4643 |
| -2.92 | 0.0056 | 0.0018 | 2.9205 | -2.42 | 0.0213 | 0.0078 | 2.4226 | -1.92 | 0.0632 | 0.0274 | 1.9305 | -1.42 | 0.1456 | 0.0778 | 1.4551 |
| -2.91 | 0.0058 | 0.0018 | 2.9105 | -2.41 | 0.0219 | 0.0080 | 2.4126 | -1.91 | 0.0644 | 0.0281 | 1.9208 | -1.41 | 0.1476 | 0.0793 | 1.4459 |
| -2.90 | 0.0060 | 0.0019 | 2.9005 | -2.40 | 0.0224 | 0.0082 | 2.4027 | -1.90 | 0.0656 | 0.0287 | 1.9111 | -1.40 | 0.1497 | 0.0808 | 1.4367 |
| -2.89 | 0.0061 | 0.0019 | 2.8906 | -2.39 | 0.0229 | 0.0084 | 2.3928 | -1.89 | 0.0669 | 0.0294 | 1.9013 | -1.39 | 0.1518 | 0.0823 | 1.4275 |
| -2.88 | 0.0063 | 0.0020 | 2.8806 | -2.38 | 0.0235 | 0.0087 | 2.3829 | -1.88 | 0.0681 | 0.0301 | 1.8916 | -1.38 | 0.1539 | 0.0838 | 1.4183 |
| -2.87 | 0.0065 | 0.0021 | 2.8706 | -2.37 | 0.0241 | 0.0089 | 2.3730 | -1.87 | 0.0694 | 0.0307 | 1.8819 | -1.37 | 0.1561 | 0.0853 | 1.4092 |
| -2.86 | 0.0067 | 0.0021 | 2.8606 | -2.36 | 0.0246 | 0.0091 | 2.3631 | -1.86 | 0.0707 | 0.0314 | 1.8723 | -1.36 | 0.1582 | 0.0869 | 1.4000 |
| -2.85 | 0.0069 | 0.0022 | 2.8506 | -2.35 | 0.0252 | 0.0094 | 2.3532 | -1.85 | 0.0721 | 0.0322 | 1.8626 | -1.35 | 0.1604 | 0.0885 | 1.3909 |
| -2.84 | 0.0071 | 0.0023 | 2.8407 | -2.34 | 0.0258 | 0.0096 | 2.3433 | -1.84 | 0.0734 | 0.0329 | 1.8529 | -1.34 | 0.1626 | 0.0901 | 1.3818 |
| -2.83 | 0.0073 | 0.0023 | 2.8307 | -2.33 | 0.0264 | 0.0099 | 2.3334 | -1.83 | 0.0748 | 0.0336 | 1.8432 | -1.33 | 0.1647 | 0.0918 | 1.3727 |
| -2.82 | 0.0075 | 0.0024 | 2.8207 | -2.32 | 0.0270 | 0.0102 | 2.3235 | -1.82 | 0.0761 | 0.0344 | 1.8336 | -1.32 | 0.1669 | 0.0934 | 1.3636 |
| -2.81 | 0.0077 | 0.0025 | 2.8107 | -2.31 | 0.0277 | 0.0104 | 2.3136 | -1.81 | 0.0775 | 0.0351 | 1.8239 | -1.31 | 0.1691 | 0.0951 | 1.3546 |
| -2.80 | 0.0079 | 0.0026 | 2.8008 | -2.30 | 0.0283 | 0.0107 | 2.3037 | -1.80 | 0.0790 | 0.0359 | 1.8143 | -1.30 | 0.1714 | 0.0968 | 1.3455 |
| -2.79 | 0.0081 | 0.0026 | 2.7908 | -2.29 | 0.0290 | 0.0110 | 2.2938 | -1.79 | 0.0804 | 0.0367 | 1.8046 | -1.29 | 0.1736 | 0.0985 | 1.3365 |
| -2.78 | 0.0084 | 0.0027 | 2.7808 | -2.28 | 0.0297 | 0.0113 | 2.2839 | -1.78 | 0.0818 | 0.0375 | 1.7950 | -1.28 | 0.1758 | 0.1003 | 1.3275 |
| -2.77 | 0.0086 | 0.0028 | 2.7708 | -2.27 | 0.0303 | 0.0116 | 2.2740 | -1.77 | 0.0833 | 0.0384 | 1.7854 | -1.27 | 0.1781 | 0.1020 | 1.3185 |
| -2.76 | 0.0088 | 0.0029 | 2.7609 | -2.26 | 0.0310 | 0.0119 | 2.2641 | -1.76 | 0.0848 | 0.0392 | 1.7758 | -1.26 | 0.1804 | 0.1038 | 1.3095 |
| -2.75 | 0.0091 | 0.0030 | 2.7509 | -2.25 | 0.0317 | 0.0122 | 2.2542 | -1.75 | 0.0863 | 0.0401 | 1.7662 | -1.25 | 0.1826 | 0.1056 | 1.3006 |
| -2.74 | 0.0093 | 0.0031 | 2.7409 | -2.24 | 0.0325 | 0.0125 | 2.2444 | -1.74 | 0.0878 | 0.0409 | 1.7566 | -1.24 | 0.1849 | 0.1075 | 1.2917 |
| -2.73 | 0.0096 | 0.0032 | 2.7310 | -2.23 | 0.0332 | 0.0129 | 2.2345 | -1.73 | 0.0893 | 0.0418 | 1.7470 | -1.23 | 0.1872 | 0.1093 | 1.2827 |
| -2.72 | 0.0099 | 0.0033 | 2.7210 | -2.22 | 0.0339 | 0.0132 | 2.2246 | -1.72 | 0.0909 | 0.0427 | 1.7374 | -1.22 | 0.1895 | 0.1112 | 1.2738 |
| -2.71 | 0.0101 | 0.0034 | 2.7110 | -2.21 | 0.0347 | 0.0136 | 2.2147 | -1.71 | 0.0925 | 0.0436 | 1.7278 | -1.21 | 0.1919 | 0.1131 | 1.2650 |
| -2.70 | 0.0104 | 0.0035 | 2.7011 | -2.20 | 0.0355 | 0.0139 | 2.2049 | -1.70 | 0.0940 | 0.0446 | 1.7183 | -1.20 | 0.1942 | 0.1151 | 1.2561 |
| -2.69 | 0.0107 | 0.0036 | 2.6911 | -2.19 | 0.0363 | 0.0143 | 2.1950 | -1.69 | 0.0957 | 0.0455 | 1.7087 | -1.19 | 0.1965 | 0.1170 | 1.2473 |
| -2.68 | 0.0110 | 0.0037 | 2.6811 | -2.18 | 0.0371 | 0.0146 | 2.1852 | -1.68 | 0.0973 | 0.0465 | 1.6992 | -1.18 | 0.1989 | 0.1190 | 1.2384 |
| -2.67 | 0.0113 | 0.0038 | 2.6712 | -2.17 | 0.0379 | 0.0150 | 2.1753 | -1.67 | 0.0989 | 0.0475 | 1.6897 | -1.17 | 0.2012 | 0.1210 | 1.2296 |
| -2.66 | 0.0116 | 0.0039 | 2.6612 | -2.16 | 0.0387 | 0.0154 | 2.1655 | -1.66 | 0.1006 | 0.0485 | 1.6801 | -1.16 | 0.2036 | 0.1230 | 1.2209 |
| -2.65 | 0.0119 | 0.0040 | 2.6512 | -2.15 | 0.0396 | 0.0158 | 2.1556 | -1.65 | 0.1023 | 0.0495 | 1.6706 | -1.15 | 0.2059 | 0.1251 | 1.2121 |
| -2.64 | 0.0122 | 0.0041 | 2.6413 | -2.14 | 0.0404 | 0.0162 | 2.1458 | -1.64 | 0.1040 | 0.0505 | 1.6611 | -1.14 | 0.2083 | 0.1271 | 1.2034 |
| -2.63 | 0.0126 | 0.0043 | 2.6313 | -2.13 | 0.0413 | 0.0166 | 2.1360 | -1.63 | 0.1057 | 0.0516 | 1.6516 | -1.13 | 0.2107 | 0.1292 | 1.1946 |
| -2.62 | 0.0129 | 0.0044 | 2.6214 | -2.12 | 0.0422 | 0.0170 | 2.1261 | -1.62 | 0.1074 | 0.0526 | 1.6422 | -1.12 | 0.2131 | 0.1314 | 1.1859 |
| -2.61 | 0.0132 | 0.0045 | 2.6114 | -2.11 | 0.0431 | 0.0174 | 2.1163 | -1.61 | 0.1092 | 0.0537 | 1.6327 | -1.11 | 0.2155 | 0.1335 | 1.1773 |
| -2.60 | 0.0136 | 0.0047 | 2.6015 | -2.10 | 0.0440 | 0.0179 | 2.1065 | -1.60 | 0.1109 | 0.0548 | 1.6232 | -1.10 | 0.2179 | 0.1357 | 1.1686 |
| -2.59 | 0.0139 | 0.0048 | 2.5915 | -2.09 | 0.0449 | 0.0183 | 2.0966 | -1.59 | 0.1127 | 0.0559 | 1.6138 | -1.09 | 0.2203 | 0.1379 | 1.1600 |
| -2.58 | 0.0143 | 0.0049 | 2.5816 | -2.08 | 0.0459 | 0.0188 | 2.0868 | -1.58 | 0.1145 | 0.0571 | 1.6044 | -1.08 | 0.2227 | 0.1401 | 1.1514 |
| -2.57 | 0.0147 | 0.0051 | 2.5716 | -2.07 | 0.0468 | 0.0192 | 2.0770 | -1.57 | 0.1163 | 0.0582 | 1.5949 | -1.07 | 0.2251 | 0.1423 | 1.1428 |
| -2.56 | 0.0151 | 0.0052 | 2.5617 | -2.06 | 0.0478 | 0.0197 | 2.0672 | -1.56 | 0.1182 | 0.0594 | 1.5855 | -1.06 | 0.2275 | 0.1446 | 1.1342 |
| -2.55 | 0.0154 | 0.0054 | 2.5517 | -2.05 | 0.0488 | 0.0202 | 2.0574 | -1.55 | 0.1200 | 0.0606 | 1.5761 | -1.05 | 0.2299 | 0.1469 | 1.1257 |
| -2.54 | 0.0158 | 0.0055 | 2.5418 | -2.04 | 0.0498 | 0.0207 | 2.0476 | -1.54 | 0.1219 | 0.0618 | 1.5667 | -1.04 | 0.2323 | 0.1492 | 1.1172 |
| -2.53 | 0.0163 | 0.0057 | 2.5318 | -2.03 | 0.0508 | 0.0212 | 2.0378 | -1.53 | 0.1238 | 0.0630 | 1.5574 | -1.03 | 0.2347 | 0.1515 | 1.1087 |
| -2.52 | 0.0167 | 0.0059 | 2.5219 | -2.02 | 0.0519 | 0.0217 | 2.0280 | -1.52 | 0.1257 | 0.0643 | 1.5480 | -1.02 | 0.2371 | 0.1539 | 1.1002 |
| -2.51 | 0.0171 | 0.0060 | 2.5119 | -2.01 | 0.0529 | 0.0222 | 2.0183 | -1.51 | 0.1276 | 0.0655 | 1.5386 | -1.01 | 0.2396 | 0.1562 | 1.0917 |

| z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) |
|-------|--------|--------|--------|-------|--------|--------|--------|------|--------|--------|--------|------|--------|--------|--------|
| -1.00 | 0.2420 | 0.1587 | 1.0833 | -0.50 | 0.3521 | 0.3085 | 0.6978 | 0.00 | 0.3989 | 0.5000 | 0.3989 | 0.50 | 0.3521 | 0.6915 | 0.1978 |
| -0.99 | 0.2444 | 0.1611 | 1.0749 | -0.49 | 0.3538 | 0.3121 | 0.6909 | 0.01 | 0.3989 | 0.5040 | 0.3940 | 0.51 | 0.3503 | 0.6950 | 0.1947 |
| -0.98 | 0.2468 | 0.1635 | 1.0665 | -0.48 | 0.3555 | 0.3156 | 0.6840 | 0.02 | 0.3989 | 0.5080 | 0.3890 | 0.52 | 0.3485 | 0.6985 | 0.1917 |
| -0.97 | 0.2492 | 0.1660 | 1.0582 | -0.47 | 0.3572 | 0.3192 | 0.6772 | 0.03 | 0.3988 | 0.5120 | 0.3841 | 0.53 | 0.3467 | 0.7019 | 0.1887 |
| -0.96 | 0.2516 | 0.1685 | 1.0499 | -0.46 | 0.3589 | 0.3228 | 0.6704 | 0.04 | 0.3986 | 0.5160 | 0.3793 | 0.54 | 0.3448 | 0.7054 | 0.1857 |
| -0.95 | 0.2541 | 0.1711 | 1.0416 | -0.45 | 0.3605 | 0.3264 | 0.6637 | 0.05 | 0.3984 | 0.5199 | 0.3744 | 0.55 | 0.3429 | 0.7088 | 0.1828 |
| -0.94 | 0.2565 | 0.1736 | 1.0333 | -0.44 | 0.3621 | 0.3300 | 0.6569 | 0.06 | 0.3982 | 0.5239 | 0.3697 | 0.56 | 0.3410 | 0.7123 | 0.1799 |
| -0.93 | 0.2589 | 0.1762 | 1.0250 | -0.43 | 0.3637 | 0.3336 | 0.6503 | 0.07 | 0.3980 | 0.5279 | 0.3649 | 0.57 | 0.3391 | 0.7157 | 0.1771 |
| -0.92 | 0.2613 | 0.1788 | 1.0168 | -0.42 | 0.3653 | 0.3372 | 0.6436 | 0.08 | 0.3977 | 0.5319 | 0.3602 | 0.58 | 0.3372 | 0.7190 | 0.1742 |
| -0.91 | 0.2637 | 0.1814 | 1.0086 | -0.41 | 0.3668 | 0.3409 | 0.6370 | 0.09 | 0.3973 | 0.5359 | 0.3556 | 0.59 | 0.3352 | 0.7224 | 0.1714 |
| -0.90 | 0.2661 | 0.1841 | 1.0004 | -0.40 | 0.3683 | 0.3446 | 0.6304 | 0.10 | 0.3970 | 0.5398 | 0.3509 | 0.60 | 0.3332 | 0.7257 | 0.1687 |
| -0.89 | 0.2685 | 0.1867 | 0.9923 | -0.39 | 0.3697 | 0.3483 | 0.6239 | 0.11 | 0.3965 | 0.5438 | 0.3464 | 0.61 | 0.3312 | 0.7291 | 0.1659 |
| -0.88 | 0.2709 | 0.1894 | 0.9842 | -0.38 | 0.3712 | 0.3520 | 0.6174 | 0.12 | 0.3961 | 0.5478 | 0.3418 | 0.62 | 0.3292 | 0.7324 | 0.1633 |
| -0.87 | 0.2732 | 0.1922 | 0.9761 | -0.37 | 0.3725 | 0.3557 | 0.6109 | 0.13 | 0.3956 | 0.5517 | 0.3373 | 0.63 | 0.3271 | 0.7357 | 0.1606 |
| -0.86 | 0.2756 | 0.1949 | 0.9680 | -0.36 | 0.3739 | 0.3594 | 0.6045 | 0.14 | 0.3951 | 0.5557 | 0.3328 | 0.64 | 0.3251 | 0.7389 | 0.1580 |
| -0.85 | 0.2780 | 0.1977 | 0.9600 | -0.35 | 0.3752 | 0.3632 | 0.5981 | 0.15 | 0.3945 | 0.5596 | 0.3284 | 0.65 | 0.3230 | 0.7422 | 0.1554 |
| -0.84 | 0.2803 | 0.2005 | 0.9520 | -0.34 | 0.3765 | 0.3669 | 0.5918 | 0.16 | 0.3939 | 0.5636 | 0.3240 | 0.66 | 0.3209 | 0.7454 | 0.1528 |
| -0.83 | 0.2827 | 0.2033 | 0.9440 | -0.33 | 0.3778 | 0.3707 | 0.5855 | 0.17 | 0.3932 | 0.5675 | 0.3197 | 0.67 | 0.3187 | 0.7486 | 0.1503 |
| -0.82 | 0.2850 | 0.2061 | 0.9360 | -0.32 | 0.3790 | 0.3745 | 0.5792 | 0.18 | 0.3925 | 0.5714 | 0.3154 | 0.68 | 0.3166 | 0.7517 | 0.1478 |
| -0.81 | 0.2874 | 0.2090 | 0.9281 | -0.31 | 0.3802 | 0.3783 | 0.5730 | 0.19 | 0.3918 | 0.5753 | 0.3111 | 0.69 | 0.3144 | 0.7549 | 0.1453 |
| -0.80 | 0.2897 | 0.2119 | 0.9202 | -0.30 | 0.3814 | 0.3821 | 0.5668 | 0.20 | 0.3910 | 0.5793 | 0.3069 | 0.70 | 0.3123 | 0.7580 | 0.1429 |
| -0.79 | 0.2920 | 0.2148 | 0.9123 | -0.29 | 0.3825 | 0.3859 | 0.5606 | 0.21 | 0.3902 | 0.5832 | 0.3027 | 0.71 | 0.3101 | 0.7611 | 0.1405 |
| -0.78 | 0.2943 | 0.2177 | 0.9045 | -0.28 | 0.3836 | 0.3897 | 0.5545 | 0.22 | 0.3894 | 0.5871 | 0.2986 | 0.72 | 0.3079 | 0.7642 | 0.1381 |
| -0.77 | 0.2966 | 0.2206 | 0.8967 | -0.27 | 0.3847 | 0.3936 | 0.5484 | 0.23 | 0.3885 | 0.5910 | 0.2944 | 0.73 | 0.3056 | 0.7673 | 0.1358 |
| -0.76 | 0.2989 | 0.2236 | 0.8889 | -0.26 | 0.3857 | 0.3974 | 0.5424 | 0.24 | 0.3876 | 0.5948 | 0.2904 | 0.74 | 0.3034 | 0.7704 | 0.1334 |
| -0.75 | 0.3011 | 0.2266 | 0.8812 | -0.25 | 0.3867 | 0.4013 | 0.5363 | 0.25 | 0.3867 | 0.5987 | 0.2863 | 0.75 | 0.3011 | 0.7734 | 0.1312 |
| -0.74 | 0.3034 | 0.2296 | 0.8734 | -0.24 | 0.3876 | 0.4052 | 0.5304 | 0.26 | 0.3857 | 0.6026 | 0.2824 | 0.76 | 0.2989 | 0.7764 | 0.1289 |
| -0.73 | 0.3056 | 0.2327 | 0.8658 | -0.23 | 0.3885 | 0.4090 | 0.5244 | 0.27 | 0.3847 | 0.6064 | 0.2784 | 0.77 | 0.2966 | 0.7794 | 0.1267 |
| -0.72 | 0.3079 | 0.2358 | 0.8581 | -0.22 | 0.3894 | 0.4129 | 0.5186 | 0.28 | 0.3836 | 0.6103 | 0.2745 | 0.78 | 0.2943 | 0.7823 | 0.1245 |
| -0.71 | 0.3101 | 0.2389 | 0.8505 | -0.21 | 0.3902 | 0.4168 | 0.5127 | 0.29 | 0.3825 | 0.6141 | 0.2706 | 0.79 | 0.2920 | 0.7852 | 0.1223 |
| -0.70 | 0.3123 | 0.2420 | 0.8429 | -0.20 | 0.3910 | 0.4207 | 0.5069 | 0.30 | 0.3814 | 0.6179 | 0.2668 | 0.80 | 0.2897 | 0.7881 | 0.1202 |
| -0.69 | 0.3144 | 0.2451 | 0.8353 | -0.19 | 0.3918 | 0.4247 | 0.5011 | 0.31 | 0.3802 | 0.6217 | 0.2630 | 0.81 | 0.2874 | 0.7910 | 0.1181 |
| -0.68 | 0.3166 | 0.2483 | 0.8278 | -0.18 | 0.3925 | 0.4286 | 0.4954 | 0.32 | 0.3790 | 0.6255 | 0.2592 | 0.82 | 0.2850 | 0.7939 | 0.1160 |
| -0.67 | 0.3187 | 0.2514 | 0.8203 | -0.17 | 0.3932 | 0.4325 | 0.4897 | 0.33 | 0.3778 | 0.6293 | 0.2555 | 0.83 | 0.2827 | 0.7967 | 0.1140 |
| -0.66 | 0.3209 | 0.2546 | 0.8128 | -0.16 | 0.3939 | 0.4364 | 0.4840 | 0.34 | 0.3765 | 0.6331 | 0.2518 | 0.84 | 0.2803 | 0.7995 | 0.1120 |
| -0.65 | 0.3230 | 0.2578 | 0.8054 | -0.15 | 0.3945 | 0.4404 | 0.4784 | 0.35 | 0.3752 | 0.6368 | 0.2481 | 0.85 | 0.2780 | 0.8023 | 0.1100 |
| -0.64 | 0.3251 | 0.2611 | 0.7980 | -0.14 | 0.3951 | 0.4443 | 0.4728 | 0.36 | 0.3739 | 0.6406 | 0.2445 | 0.86 | 0.2756 | 0.8051 | 0.1080 |
| -0.63 | 0.3271 | 0.2643 | 0.7906 | -0.13 | 0.3956 | 0.4483 | 0.4673 | 0.37 | 0.3725 | 0.6443 | 0.2409 | 0.87 | 0.2732 | 0.8078 | 0.1061 |
| -0.62 | 0.3292 | 0.2676 | 0.7833 | -0.12 | 0.3961 | 0.4522 | 0.4618 | 0.38 | 0.3712 | 0.6480 | 0.2374 | 0.88 | 0.2709 | 0.8106 | 0.1042 |
| -0.61 | 0.3312 | 0.2709 | 0.7759 | -0.11 | 0.3965 | 0.4562 | 0.4564 | 0.39 | 0.3697 | 0.6517 | 0.2339 | 0.89 | 0.2685 | 0.8133 | 0.1023 |
| -0.60 | 0.3332 | 0.2743 | 0.7687 | -0.10 | 0.3970 | 0.4602 | 0.4509 | 0.40 | 0.3683 | 0.6554 | 0.2304 | 0.90 | 0.2661 | 0.8159 | 0.1004 |
| -0.59 | 0.3352 | 0.2776 | 0.7614 | -0.09 | 0.3973 | 0.4641 | 0.4456 | 0.41 | 0.3668 | 0.6591 | 0.2270 | 0.91 | 0.2637 | 0.8186 | 0.0986 |
| -0.58 | 0.3372 | 0.2810 | 0.7542 | -0.08 | 0.3977 | 0.4681 | 0.4402 | 0.42 | 0.3653 | 0.6628 | 0.2236 | 0.92 | 0.2613 | 0.8212 | 0.0968 |
| -0.57 | 0.3391 | 0.2843 | 0.7471 | -0.07 | 0.3980 | 0.4721 | 0.4349 | 0.43 | 0.3637 | 0.6664 | 0.2203 | 0.93 | 0.2589 | 0.8238 | 0.0950 |
| -0.56 | 0.3410 | 0.2877 | 0.7399 | -0.06 | 0.3982 | 0.4761 | 0.4297 | 0.44 | 0.3621 | 0.6700 | 0.2169 | 0.94 | 0.2565 | 0.8264 | 0.0933 |
| -0.55 | 0.3429 | 0.2912 | 0.7328 | -0.05 | 0.3984 | 0.4801 | 0.4244 | 0.45 | 0.3605 | 0.6736 | 0.2137 | 0.95 | 0.2541 | 0.8289 | 0.0916 |
| -0.54 | 0.3448 | 0.2946 | 0.7257 | -0.04 | 0.3986 | 0.4840 | 0.4193 | 0.46 | 0.3589 | 0.6772 | 0.2104 | 0.96 | 0.2516 | 0.8315 | 0.0899 |
| -0.53 | 0.3467 | 0.2981 | 0.7187 | -0.03 | 0.3988 | 0.4880 | 0.4141 | 0.47 | 0.3572 | 0.6808 | 0.2072 | 0.97 | 0.2492 | 0.8340 | 0.0882 |
| -0.52 | 0.3485 | 0.3015 | 0.7117 | -0.02 | 0.3989 | 0.4920 | 0.4090 | 0.48 | 0.3555 | 0.6844 | 0.2040 | 0.98 | 0.2468 | 0.8365 | 0.0865 |
| -0.51 | 0.3503 | 0.3050 | 0.7047 | -0.01 | 0.3989 | 0.4960 | 0.4040 | 0.49 | 0.3538 | 0.6879 | 0.2009 | 0.99 | 0.2444 | 0.8389 | 0.0849 |

| z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) | z | f(z) | F(z) | L(z) |
|------|--------|--------|--------|------|--------|--------|--------|------|--------|--------|--------|------|--------|--------|--------|
| 1.00 | 0.2420 | 0.8413 | 0.0833 | 1.50 | 0.1295 | 0.9332 | 0.0293 | 2.00 | 0.0540 | 0.9772 | 0.0085 | 2.50 | 0.0175 | 0.9938 | 0.0020 |
| 1.01 | 0.2396 | 0.8438 | 0.0817 | 1.51 | 0.1276 | 0.9345 | 0.0286 | 2.01 | 0.0529 | 0.9778 | 0.0083 | 2.51 | 0.0171 | 0.9940 | 0.0019 |
| 1.02 | 0.2371 | 0.8461 | 0.0802 | 1.52 | 0.1257 | 0.9357 | 0.0280 | 2.02 | 0.0519 | 0.9783 | 0.0080 | 2.52 | 0.0167 | 0.9941 | 0.0019 |
| 1.03 | 0.2347 | 0.8485 | 0.0787 | 1.53 | 0.1238 | 0.9370 | 0.0274 | 2.03 | 0.0508 | 0.9788 | 0.0078 | 2.53 | 0.0163 | 0.9943 | 0.0018 |
| 1.04 | 0.2323 | 0.8508 | 0.0772 | 1.54 | 0.1219 | 0.9382 | 0.0267 | 2.04 | 0.0498 | 0.9793 | 0.0076 | 2.54 | 0.0158 | 0.9945 | 0.0018 |
| 1.05 | 0.2299 | 0.8531 | 0.0757 | 1.55 | 0.1200 | 0.9394 | 0.0261 | 2.05 | 0.0488 | 0.9798 | 0.0074 | 2.55 | 0.0154 | 0.9946 | 0.0017 |
| 1.06 | 0.2275 | 0.8554 | 0.0742 | 1.56 | 0.1182 | 0.9406 | 0.0255 | 2.06 | 0.0478 | 0.9803 | 0.0072 | 2.56 | 0.0151 | 0.9948 | 0.0017 |
| 1.07 | 0.2251 | 0.8577 | 0.0728 | 1.57 | 0.1163 | 0.9418 | 0.0249 | 2.07 | 0.0468 | 0.9808 | 0.0070 | 2.57 | 0.0147 | 0.9949 | 0.0016 |
| 1.08 | 0.2227 | 0.8599 | 0.0714 | 1.58 | 0.1145 | 0.9429 | 0.0244 | 2.08 | 0.0459 | 0.9812 | 0.0068 | 2.58 | 0.0143 | 0.9951 | 0.0016 |
| 1.09 | 0.2203 | 0.8621 | 0.0700 | 1.59 | 0.1127 | 0.9441 | 0.0238 | 2.09 | 0.0449 | 0.9817 | 0.0066 | 2.59 | 0.0139 | 0.9952 | 0.0015 |
| 1.10 | 0.2179 | 0.8643 | 0.0686 | 1.60 | 0.1109 | 0.9452 | 0.0232 | 2.10 | 0.0440 | 0.9821 | 0.0065 | 2.60 | 0.0136 | 0.9953 | 0.0015 |
| 1.11 | 0.2155 | 0.8665 | 0.0673 | 1.61 | 0.1092 | 0.9463 | 0.0227 | 2.11 | 0.0431 | 0.9826 | 0.0063 | 2.61 | 0.0132 | 0.9955 | 0.0014 |
| 1.12 | 0.2131 | 0.8686 | 0.0659 | 1.62 | 0.1074 | 0.9474 | 0.0222 | 2.12 | 0.0422 | 0.9830 | 0.0061 | 2.62 | 0.0129 | 0.9956 | 0.0014 |
| 1.13 | 0.2107 | 0.8708 | 0.0646 | 1.63 | 0.1057 | 0.9484 | 0.0216 | 2.13 | 0.0413 | 0.9834 | 0.0060 | 2.63 | 0.0126 | 0.9957 | 0.0013 |
| 1.14 | 0.2083 | 0.8729 | 0.0634 | 1.64 | 0.1040 | 0.9495 | 0.0211 | 2.14 | 0.0404 | 0.9838 | 0.0058 | 2.64 | 0.0122 | 0.9959 | 0.0013 |
| 1.15 | 0.2059 | 0.8749 | 0.0621 | 1.65 | 0.1023 | 0.9505 | 0.0206 | 2.15 | 0.0396 | 0.9842 | 0.0056 | 2.65 | 0.0119 | 0.9960 | 0.0012 |
| 1.16 | 0.2036 | 0.8770 | 0.0609 | 1.66 | 0.1006 | 0.9515 | 0.0201 | 2.16 | 0.0387 | 0.9846 | 0.0055 | 2.66 | 0.0116 | 0.9961 | 0.0012 |
| 1.17 | 0.2012 | 0.8790 | 0.0596 | 1.67 | 0.0989 | 0.9525 | 0.0197 | 2.17 | 0.0379 | 0.9850 | 0.0053 | 2.67 | 0.0113 | 0.9962 | 0.0012 |
| 1.18 | 0.1989 | 0.8810 | 0.0584 | 1.68 | 0.0973 | 0.9535 | 0.0192 | 2.18 | 0.0371 | 0.9854 | 0.0052 | 2.68 | 0.0110 | 0.9963 | 0.0011 |
| 1.19 | 0.1965 | 0.8830 | 0.0573 | 1.69 | 0.0957 | 0.9545 | 0.0187 | 2.19 | 0.0363 | 0.9857 | 0.0050 | 2.69 | 0.0107 | 0.9964 | 0.0011 |
| 1.20 | 0.1942 | 0.8849 | 0.0561 | 1.70 | 0.0940 | 0.9554 | 0.0183 | 2.20 | 0.0355 | 0.9861 | 0.0049 | 2.70 | 0.0104 | 0.9965 | 0.0011 |
| 1.21 | 0.1919 | 0.8869 | 0.0550 | 1.71 | 0.0925 | 0.9564 | 0.0178 | 2.21 | 0.0347 | 0.9864 | 0.0047 | 2.71 | 0.0101 | 0.9966 | 0.0010 |
| 1.22 | 0.1895 | 0.8888 | 0.0538 | 1.72 | 0.0909 | 0.9573 | 0.0174 | 2.22 | 0.0339 | 0.9868 | 0.0046 | 2.72 | 0.0099 | 0.9967 | 0.0010 |
| 1.23 | 0.1872 | 0.8907 | 0.0527 | 1.73 | 0.0893 | 0.9582 | 0.0170 | 2.23 | 0.0332 | 0.9871 | 0.0045 | 2.73 | 0.0096 | 0.9968 | 0.0010 |
| 1.24 | 0.1849 | 0.8925 | 0.0517 | 1.74 | 0.0878 | 0.9591 | 0.0166 | 2.24 | 0.0325 | 0.9875 | 0.0044 | 2.74 | 0.0093 | 0.9969 | 0.0009 |
| 1.25 | 0.1826 | 0.8944 | 0.0506 | 1.75 | 0.0863 | 0.9599 | 0.0162 | 2.25 | 0.0317 | 0.9878 | 0.0042 | 2.75 | 0.0091 | 0.9970 | 0.0009 |
| 1.26 | 0.1804 | 0.8962 | 0.0495 | 1.76 | 0.0848 | 0.9608 | 0.0158 | 2.26 | 0.0310 | 0.9881 | 0.0041 | 2.76 | 0.0088 | 0.9971 | 0.0009 |
| 1.27 | 0.1781 | 0.8980 | 0.0485 | 1.77 | 0.0833 | 0.9616 | 0.0154 | 2.27 | 0.0303 | 0.9884 | 0.0040 | 2.77 | 0.0086 | 0.9972 | 0.0008 |
| 1.28 | 0.1758 | 0.8997 | 0.0475 | 1.78 | 0.0818 | 0.9625 | 0.0150 | 2.28 | 0.0297 | 0.9887 | 0.0039 | 2.78 | 0.0084 | 0.9973 | 0.0008 |
| 1.29 | 0.1736 | 0.9015 | 0.0465 | 1.79 | 0.0804 | 0.9633 | 0.0146 | 2.29 | 0.0290 | 0.9890 | 0.0038 | 2.79 | 0.0081 | 0.9974 | 0.0008 |
| 1.30 | 0.1714 | 0.9032 | 0.0455 | 1.80 | 0.0790 | 0.9641 | 0.0143 | 2.30 | 0.0283 | 0.9893 | 0.0037 | 2.80 | 0.0079 | 0.9974 | 0.0008 |
| 1.31 | 0.1691 | 0.9049 | 0.0446 | 1.81 | 0.0775 | 0.9649 | 0.0139 | 2.31 | 0.0277 | 0.9896 | 0.0036 | 2.81 | 0.0077 | 0.9975 | 0.0007 |
| 1.32 | 0.1669 | 0.9066 | 0.0436 | 1.82 | 0.0761 | 0.9656 | 0.0136 | 2.32 | 0.0270 | 0.9898 | 0.0035 | 2.82 | 0.0075 | 0.9976 | 0.0007 |
| 1.33 | 0.1647 | 0.9082 | 0.0427 | 1.83 | 0.0748 | 0.9664 | 0.0132 | 2.33 | 0.0264 | 0.9901 | 0.0034 | 2.83 | 0.0073 | 0.9977 | 0.0007 |
| 1.34 | 0.1626 | 0.9099 | 0.0418 | 1.84 | 0.0734 | 0.9671 | 0.0129 | 2.34 | 0.0258 | 0.9904 | 0.0033 | 2.84 | 0.0071 | 0.9977 | 0.0007 |
| 1.35 | 0.1604 | 0.9115 | 0.0409 | 1.85 | 0.0721 | 0.9678 | 0.0126 | 2.35 | 0.0252 | 0.9906 | 0.0032 | 2.85 | 0.0069 | 0.9978 | 0.0006 |
| 1.36 | 0.1582 | 0.9131 | 0.0400 | 1.86 | 0.0707 | 0.9686 | 0.0123 | 2.36 | 0.0246 | 0.9909 | 0.0031 | 2.86 | 0.0067 | 0.9979 | 0.0006 |
| 1.37 | 0.1561 | 0.9147 | 0.0392 | 1.87 | 0.0694 | 0.9693 | 0.0119 | 2.37 | 0.0241 | 0.9911 | 0.0030 | 2.87 | 0.0065 | 0.9979 | 0.0006 |
| 1.38 | 0.1539 | 0.9162 | 0.0383 | 1.88 | 0.0681 | 0.9699 | 0.0116 | 2.38 | 0.0235 | 0.9913 | 0.0029 | 2.88 | 0.0063 | 0.9980 | 0.0006 |
| 1.39 | 0.1518 | 0.9177 | 0.0375 | 1.89 | 0.0669 | 0.9706 | 0.0113 | 2.39 | 0.0229 | 0.9916 | 0.0028 | 2.89 | 0.0061 | 0.9981 | 0.0006 |
| 1.40 | 0.1497 | 0.9192 | 0.0367 | 1.90 | 0.0656 | 0.9713 | 0.0111 | 2.40 | 0.0224 | 0.9918 | 0.0027 | 2.90 | 0.0060 | 0.9981 | 0.0005 |
| 1.41 | 0.1476 | 0.9207 | 0.0359 | 1.91 | 0.0644 | 0.9719 | 0.0108 | 2.41 | 0.0219 | 0.9920 | 0.0026 | 2.91 | 0.0058 | 0.9982 | 0.0005 |
| 1.42 | 0.1456 | 0.9222 | 0.0351 | 1.92 | 0.0632 | 0.9726 | 0.0105 | 2.42 | 0.0213 | 0.9922 | 0.0026 | 2.92 | 0.0056 | 0.9982 | 0.0005 |
| 1.43 | 0.1435 | 0.9236 | 0.0343 | 1.93 | 0.0620 | 0.9732 | 0.0102 | 2.43 | 0.0208 | 0.9925 | 0.0025 | 2.93 | 0.0055 | 0.9983 | 0.0005 |
| 1.44 | 0.1415 | 0.9251 | 0.0336 | 1.94 | 0.0608 | 0.9738 | 0.0100 | 2.44 | 0.0203 | 0.9927 | 0.0024 | 2.94 | 0.0053 | 0.9984 | 0.0005 |
| 1.45 | 0.1394 | 0.9265 | 0.0328 | 1.95 | 0.0596 | 0.9744 | 0.0097 | 2.45 | 0.0198 | 0.9929 | 0.0023 | 2.95 | 0.0051 | 0.9984 | 0.0005 |
| 1.46 | 0.1374 | 0.9279 | 0.0321 | 1.96 | 0.0584 | 0.9750 | 0.0094 | 2.46 | 0.0194 | 0.9931 | 0.0023 | 2.96 | 0.0050 | 0.9985 | 0.0004 |
| 1.47 | 0.1354 | 0.9292 | 0.0314 | 1.97 | 0.0573 | 0.9756 | 0.0092 | 2.47 | 0.0189 | 0.9933 | 0.0022 | 2.97 | 0.0048 | 0.9985 | 0.0004 |
| 1.48 | 0.1334 | 0.9306 | 0.0307 | 1.98 | 0.0562 | 0.9761 | 0.0090 | 2.48 | 0.0184 | 0.9934 | 0.0021 | 2.98 | 0.0047 | 0.9986 | 0.0004 |
| 1.49 | 0.1315 | 0.9319 | 0.0300 | 1.99 | 0.0551 | 0.9767 | 0.0087 | 2.49 | 0.0180 | 0.9936 | 0.0021 | 2.99 | 0.0046 | 0.9986 | 0.0004 |