Primer on Inventory Management – Solutions

Continuous Review Model

Solution to Exercise 1:

i. Annual demand is 200 units.

Iteration 0:

$$x_0 = \sqrt{\frac{2\mu K}{h}} = \sqrt{\frac{2 \cdot 200 \cdot 50}{0.2 \cdot 10}} = 100 \tag{67}$$

$$r_0 = F_{LT}^{-1}\left(1 - \frac{x_0 \cdot h}{\mu \cdot p}\right) = F_{LT}^{-1}\left(1 - \frac{100 \cdot 2}{200 \cdot 25}\right) = F_{LT}^{-1}(0.96) = 100 + 1.75 \cdot 25 = 144 \tag{68}$$

Iteration 1:

$$x_1 = \sqrt{\frac{2\mu}{h}\big(K + p \cdot L\left(\frac{r_0 - \mu_{LT}}{\sigma_{LT}}\right)\sigma_{LT}\big)} \tag{69}$$

$$= \sqrt{\frac{2 \cdot 200}{2}\big(50 + 25 \cdot L\left(\frac{144 - 100}{25}\right) \cdot 25\big)} \tag{70}$$

$$= \sqrt{200\big(50 + 25 \cdot 0.0162 \cdot 25\big)} \tag{71}$$

$$= 110 \tag{72}$$

$$r_1 = F_{LT}^{-1}\left(1 - \frac{x_1 \cdot h}{\mu \cdot p}\right) \tag{73}$$

$$= F_{LT}^{-1}\left(1 - \frac{110 \cdot 2}{200 \cdot 25}\right) \tag{74}$$

$$= F_{LT}^{-1}(0.956) \tag{75}$$

$$= 100 + 1.70 \cdot 25 \tag{76}$$

$$= 142.5 \tag{77}$$

Iteration 2:

$$x_1 = \sqrt{\frac{2 \cdot 200}{2}\left(50 + 25 \cdot L\left(\frac{142.5 - 100}{25}\right) \cdot 25\right)} \tag{78}$$

$$= \sqrt{200\left(50 + 25 \cdot 0.0183 \cdot 25\right)} \tag{79}$$

$$= 110.85 \tag{80}$$

$$r_1 = F_{LT}^{-1}\left(1 - \frac{110.85 \cdot 2}{200 \cdot 25}\right) \tag{81}$$

$$= F_{LT}^{-1}(0.95566) \tag{82}$$

$$= 100 + 1.7024 \cdot 25 \tag{83}$$

$$= 142.56 \tag{84}$$

Since $(142.56 - 142.5)/142.5 = 0.042\% < 0.1\%$ we stop with $x^\star = 111$ and $r^\star = 143$. Note, that we round, since we can only consider integral numbers of jars.

ii. In general, the expected cost are computed as

$$Z(x, r) \approx h \cdot \left(r - \mu_{LT} + \frac{x}{2}\right) + p \cdot \frac{\mu}{x} \cdot L\left(\frac{r - \mu_{LT}}{\sigma_{LT}}\right) \cdot \sigma_{LT} + K \cdot \frac{\mu}{x} \tag{85}$$

For $x^\star = 111$ and $r^\star = 143$ we obtain

$$Z(111, 143) \approx 2 \cdot \left(143 - 100 + \frac{111}{2}\right) + 25 \cdot \frac{200}{111} \cdot L\left(\frac{143 - 100}{25}\right) \cdot 25 + 50 \cdot \frac{200}{111} \tag{86}$$

$$= 307.70 \tag{87}$$

iii. We have already computed the EOQ and its corresponding re-order point in Iteration 0. The expected cost for $x^0 = 100$ and $r^0 = 144$ is

$$Z(100, 144) \approx 2 \cdot \left(144 - 100 + \frac{100}{2}\right) + 25 \cdot \frac{200}{100} \cdot L\left(\frac{144 - 100}{25}\right) \cdot 25 + 50 \cdot \frac{200}{100} \tag{88}$$

$$= 188 + 19.75 + 100 \tag{89}$$

$$= 307.75 \tag{90}$$

The cost of this solution is already very close to the cost of the optimal solution.

Solution to Exercise 2:

i. The cost function assumes, that the average inventory level *on-hand* is equal to $r - \mu_{LT} + x/2$. In fact, this is expected value considers both, positive and negative inventory levels and, hence, undererstimates the real expected *on-hand* inventory level. It is reasonalby accurate for situations with high service levels and/or backorder penalty cost and therefore very rare backorders.

ii. The mathematical programm with $p = 0$ reads

$$\begin{aligned} \min_{x,r} \quad & c \cdot \mu + h \cdot \left(r - \mu_{LT} + \frac{x}{2} \right) + K \cdot \frac{\mu}{x} \\ \text{s.t.} \quad & F_{LT}(r) \geq \alpha \end{aligned}$$

We note, that the constraint does only depend on r. Further observe that the objective function and the constraint are both non-decreasing in r. In our case, this means that smaller values for r will lead to lower cost and a lower α-SL. It is therefore optimal to first determine the smallest $r = r^\star$ that datisfies the constraint and next minimize the optjective function for a given $r^\star$, i.e.

$$\begin{aligned} & \min_{x} \quad c \cdot \mu + h \cdot \left(r^\star - \mu_{LT} + \frac{x}{2} \right) + K \cdot \frac{\mu}{x} \\ = & \quad c \cdot \mu + h \cdot (r^\star - \mu_{LT}) + \min_{x} \left(K \cdot \frac{\mu}{x} + \frac{x}{2} \cdot h \right) \end{aligned}$$

We see, that the part of the function, that depends on x is equal to the objective function of the EOQ model! The optimal solution for $x^\star$ is therefore given by the EOQ formula.

iii. The mathematical programm with $p = 0$ reads

$$\begin{aligned} \min_{x,r} \quad & c \cdot \mu + h \cdot \left(r - \mu_{LT} + \frac{x}{2} \right) + K \cdot \frac{\mu}{x} \\ \text{s.t.} \quad & 1 - \frac{\sigma_{LT}}{x} \cdot L\left(\frac{r - \mu_{LT}}{\sigma_{LT}} \right) \geq \beta \end{aligned}$$

Obviously this program is much more complicated than the program with an α-service-level constraint. Most strikingly, the constraint involves both variables. Additionally, the loss function is decreasing in r.